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Theoretical, Experimental, and Computational Evaluation of Disk-Loaded Circular Wave Guides

Thomas M. Wallett and A. Haq Qureshi
Lewis Research Center
Cleveland, Ohio

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THEORETICAL, EXPERIMENTAL, AND COMPUTATIONAL EVALUATION OF DISK-LOADED CIRCULAR WAVE GUIDES

Thomas Michael Wallett and A. Haq Qureshi*
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

SUMMARY

A disk-loaded circular wave guide structure and test fixture were fabricated. The dispersion characteristics were found by theoretical analysis, experimental testing, and computer simulation using the codes ARGUS and SOS. Interaction impedances were computed based on the corresponding dispersion characteristics. Finally, an equivalent circuit model for one period of the structure was chosen using equivalent circuit models for cylindrical wave guides of different radii. Optimum values for the discrete capacitors and inductors describing discontinuities between cylindrical wave guides were found using the computer code TOUCHSTONE.

INTRODUCTION

The disk-loaded circular wave guide structure is a forward-wave type structure and has applications in linear accelerators, high power TWTs, and millimeter-wave devices [1]. The theoretical analysis of such a structure can be accomplished rigorously by using a generalized mode matching technique. Scattering parameters for circular wave guide junctions have been calculated using a computer program based on this type of technique [2].

Another effective approach based on a TM_{01} field analysis [3,4] has been done to determine useful parameters such as power flow, group velocity, and interaction impedance for this type of structure. One of the striking characteristics of these structures is the high gain per unit length, nearly twice that of a helix structure with the same beam hole size and the same beam velocity [3]. The disk-loaded circular wave guide structure, however, is limited in bandwidth when compared to the helix structure and would be most suited to high-gain, low power applications such as in a miniature traveling-wave device [3].

THEORETICAL EVALUATION

The disk hole radius, the cylindrical wave guide radius, the slot length, and the disk thickness are represented by a, b, s and t, respectively (Figure 1).

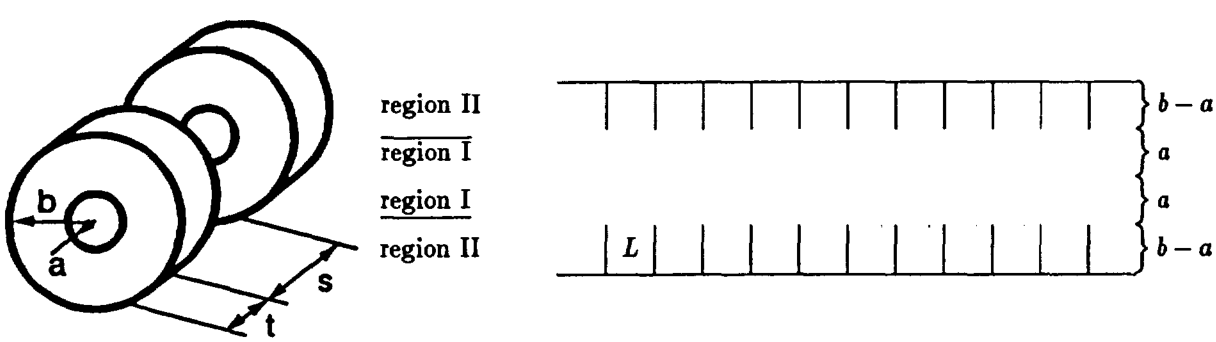

Figure 1 - Disk-loaded circular wave guide with dimensions.

*National Research Council—NASA Research Associate at Lewis Research Center.

Using cylindrical coordinates, the fields for $k < \beta$ in the region $\rho < a$ are represented by

$$E_z = E_0 \mathrm{J}_0(\gamma\rho)e^{-j\beta z} = E_0 \mathrm{I}_0(\tau\rho)e^{-j\beta z}, \tag{1}$$

$$E_\rho = jE_0(\beta/\gamma)\mathrm{J}_1(\gamma\rho)e^{-j\beta z} = jE_0(\beta/\tau)\mathrm{I}_1(\tau\rho)e^{-j\beta z}, \tag{2}$$

$$H_\phi = j(E_0/\eta)(k/\gamma)\mathrm{J}_1(\gamma\rho)e^{-j\beta z} = j(E_0/\eta)(k/\tau)\mathrm{I}_1(\tau\rho)e^{-j\beta z}. \tag{3}$$

They are assumed to be finite, axially symmetric, and propagating in the z-direction. The parameters γ, β, and k are the radial, axial, and free space propagation constants with $\gamma^2 = k^2 - \beta^2 = -\tau^2$, and $\eta = (\mu_0/\epsilon_0)^{1/2} = 377\ \Omega$. J_0 and J_1 represent zero and first order Bessel functions while I_0 and I_1 represent zero and first order modified Bessel functions.

In the region $a < \rho < b$ we assume that the electric field is TEM_ρ with the guide wavelength much greater than the period L such that $E_z = 0$ at $\rho = b$.

$$E_z \propto [\mathrm{J}_0(k\rho)\mathrm{N}_0(kb) - \mathrm{N}_0(k\rho)\mathrm{J}_0(kb)]e^{-j\beta z}, \tag{4}$$

$$E_\phi = 0, \tag{5}$$

$$H_\phi \propto (j/\eta)[\mathrm{J}_1(k\rho)\mathrm{N}_0(kb) - \mathrm{N}_1(k\rho)\mathrm{J}_0(kb)]e^{-j\beta z}, \tag{6}$$

Again, J_0 and J_1 represent zero and first order Bessel functions while N_0 and N_1 represent zero and first order Neumann functions, respectively. H_ϕ is found from $\vec{\nabla} \times \vec{E} = -j\omega\mu_0\vec{H}$.

Matching radial admittances (H_ϕ/E_z) across the slot length in both regions results in the dispersion relation for the disk-loaded structure as given by Chu and Hansen [4]

$$\frac{1}{\gamma a}\frac{\mathrm{J}_1(\gamma a)}{\mathrm{J}_0(\gamma a)} = \frac{1}{\tau a}\frac{\mathrm{I}_1(\tau a)}{\mathrm{I}_0(\tau a)} = \frac{1}{1-\eta}\frac{1}{ka}\frac{\mathrm{J}_1(ka)\mathrm{N}_0(kb) - \mathrm{N}_1(ka)\mathrm{J}_0(kb)}{\mathrm{J}_0(ka)\mathrm{N}_0(kb) - \mathrm{N}_0(ka)\mathrm{J}_0(kb)}. \tag{7}$$

The term $1/(1-\eta)$ is an empirical correction to the zero disk thickness result. It involves the spatial factor $\eta = t/(t+s)$ which takes into account the fact that the electric field E_z is zero at the disk edge. The electric field across the slot at $\rho = a$ is assumed to be constant in the z-direction.

Taking the derivative from the above dispersion relation with respect to k we find

$$\frac{\mathrm{d}\beta}{\mathrm{d}k} = \frac{k}{\beta} - \frac{\gamma}{\beta}\frac{\mathrm{d}\alpha(k)/\mathrm{d}k}{\mathrm{d}\phi(\gamma)/\mathrm{d}\gamma} = \frac{k}{\beta} + \frac{\tau}{\beta}\frac{\mathrm{d}\alpha(k)/\mathrm{d}k}{\mathrm{d}\phi(\tau)/\mathrm{d}\tau} \tag{8}$$

where

$$\phi(\gamma) = \frac{1}{\gamma a}\frac{\mathrm{J}_1(\gamma a)}{\mathrm{J}_0(\gamma a)} = \phi(\tau) = \frac{1}{\tau a}\frac{\mathrm{I}_1(\tau a)}{\mathrm{I}_0(\tau a)} \tag{9}$$

and

$$\alpha(k) = \frac{1}{1-\eta}\frac{1}{ka}\frac{\mathrm{J}_1(ka)\mathrm{N}_0(kb) - \mathrm{N}_1(ka)\mathrm{J}_0(kb)}{\mathrm{J}_0(ka)\mathrm{N}_0(kb) - \mathrm{N}_0(ka)\mathrm{J}_0(kb)}. \tag{10}$$

The group velocity can be obtained using $\mathrm{d}\beta/\mathrm{d}k$ and is given by

$$v_g = \frac{1}{(\mu_0\epsilon_0)^{1/2}}\frac{1}{\mathrm{d}\beta/\mathrm{d}k}. \tag{11}$$

The low frequency cutoff of the disk-loaded wave guide is simply that of the cylindrical wave guide without disks. It is determined from the condition $J_0(kb) = 0$ which yields $kb \approx 2.405$. This represents a wavelength $\lambda_{lc} \approx 2\pi b/2.405$. The high frequency cutoff of the structure depends on the disk hole size provided the period $L < (b-a)/2$. This cutoff frequency is determined from the condition $J_1(ka)N_0(kb) - N_1(ka)J_0(kb) = 0$. The wavelength which corresponds to the high frequency cutoff is approximately given by $\lambda \approx 4(b-a)$. If $L > (b-a)/2$ then the high frequency cutoff is determined solely by the period of the disks and $\lambda = 2L$.

The appropriate expressions for the electric and magnetic field intensities, equations (2) and (3), are used to evaluate the time-average power flow through the disk-loaded structure.

$$\overline{P} = \frac{1}{2}\,\Re e\Big\{\int_S (\vec{E}\times\vec{H}^*)\cdot d\vec{S}\Big\} = \frac{1}{2}\int_0^{2\pi}\int_0^{a} E_\rho H_\phi\,\rho\,d\rho\,d\phi$$

$$\overline{P} = \pi\,\frac{E_0^{\,2}}{\eta}\,\frac{\beta k}{\tau^2}\int_0^{a} I_1^{\,2}(\tau\rho)\rho\,d\rho = \pi\,\frac{E_0^{\,2}}{\eta}\,\frac{\beta k}{\tau^4}\int_0^{\gamma a} I_1^{\,2}(\tau a)\,\tau\cdot\rho\,d(\tau\rho)$$

$$\overline{P} = \frac{E_0^{\,2}}{\eta}\,\frac{\pi}{2}\,\frac{\beta k}{\tau^2}\,a^2[I_1^{\,2}(\tau a) - I_0(\tau a)\,I_2(\tau a)]. \tag{12}$$

Using the expressions for E_z and $\overline{P}$, equations (1) and (12), respectively, the interaction impedance is found to be

$$Z(\rho) = \frac{|E_z^{\,2}(\rho)|}{2\beta^2\,\overline{P}} = \frac{\eta(\tau/\beta)(\tau/k)}{\pi\beta^2 a^2}\,\frac{I_0^{\,2}(\tau\rho)}{[I_1^{\,2}(\tau a) - I_0(\tau a)I_2(\tau a)]}. \tag{13}$$

The average interaction impedance is found to be

$$\overline{Z}(\rho) = \frac{\int |E_z^{\,2}(\rho)|\,dS}{2\beta^2\,\overline{P}S} = \frac{\eta(\tau/\beta)(\tau/k)}{\pi\beta^2 a^2}\,\frac{I_0^{\,2}(\tau\rho) - I_1^{\,2}(\tau\rho)}{[I_1^{\,2}(\tau a) - I_0(\tau a)I_2(\tau a)]}. \tag{14}$$

The area of integration is over the cross-sectional area S of the electron beam [5]. For the calculation of average interaction impedance, the cross-sectional area $S = \pi r^2$ was that of a perturbing dielectric rod of radius $r = 0.0155$ inch used in the experimental evaluation of the structure.

Space harmonics for the axial field are ignored in this analysis but can be found for the z-component of the electric field.

$$E_z(\rho,z) = \sum_{n=-\infty}^{\infty} A_n I_0(\tau_n\rho)e^{-j\beta_n z} \tag{15}$$

with $\tau_n^{\,2} = \beta_n^{\,2} - k^2$ and $\beta_n = \beta_0 + 2\pi n/L$. Assuming a uniform electric field E_a at $\rho = a$ across a slot of width l, the coefficients A_n can be found from

$$E_z(a,z) = \sum_{n=-\infty}^{\infty} A_n I_0(\tau_n a)e^{-j\beta_n z} = E_a. \tag{16}$$

Actually the electric field at $\rho = a$ across the slot varies approximately as $\cosh(mz)$ where m is experimentally determined [6].

Multiplying by $e^{j\beta_m z}$ and integrating over the slot we have

$$A_m I_0(\tau_m a) = \frac{1}{L}\int_{-l/2}^{+l/2} E_a e^{j\beta_m z}\, dz$$

or

$$A_m = E_a \frac{l/L}{I_0(\tau_m a)} \frac{\sin(\beta_m l/2)}{\beta_m l/2} e^{j(\beta_m l/2)}. \tag{17}$$

Equation (7) was evaluated to obtain the theoretical dispersion characteristics for structures with 0.003, 0.005, 0.008, 0.010, and 0.020 inch thick disks and slot lengths of 0.16 inch (Figure 2).

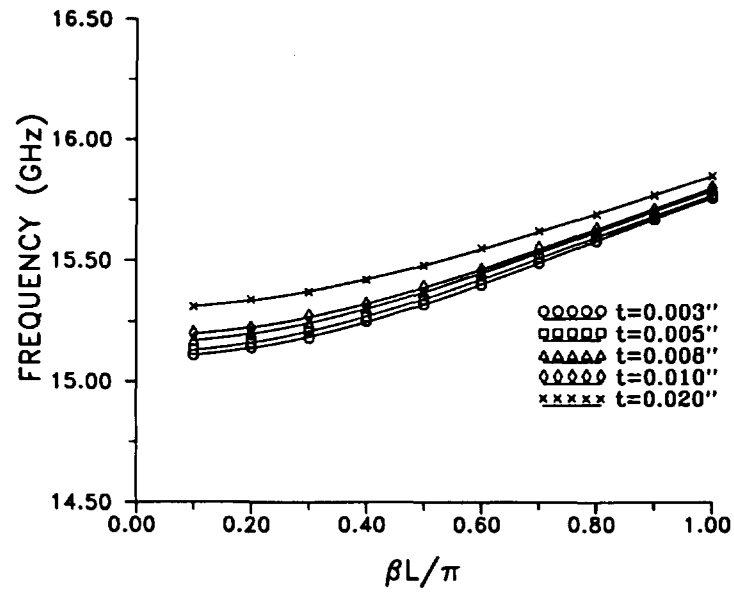

Figure 2 - Theoretical dispersion characteristics of the disk-loaded structures.

Equation (14) was then evaluated to find the theoretical interaction impedances (Figure 3).

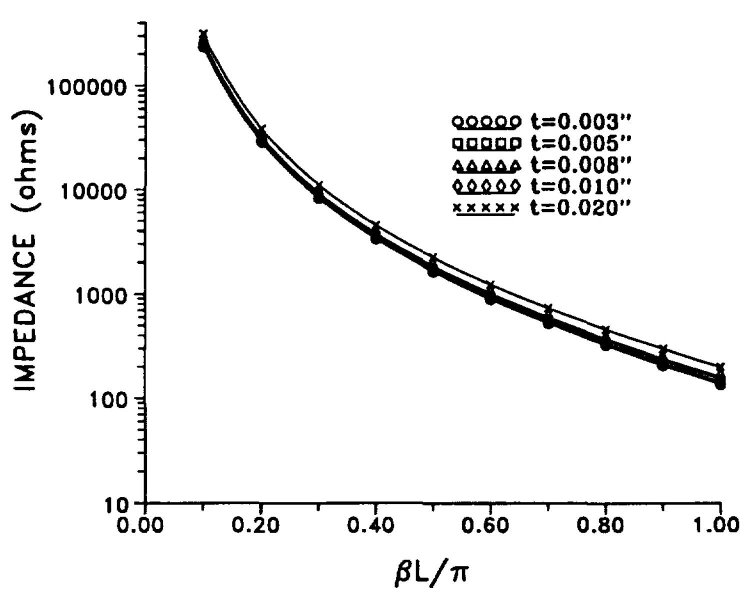

Figure 3 - Theoretical interaction impedances of the disk-loaded structures.

EXPERIMENTAL EVALUATION

In many instances the theoretical analysis of a circuit may be too complicated to solve for the dispersion relation of a structure. The dispersion characteristics can be experimentally measured using a standard technique [1,7] in which resonance frequencies of a section of the circuit are measured with and without a perturbing dielectric rod of radius r placed along the axis of symmetry. In the present case, the objective was to verify equations (7) and (14) with the experimental results as well as test the utility of the available computer codes, ARGUS [8] and SOS [9].

A circuit with period L and N periods in length is shorted at planes of mirror symmetry (Figure 4).

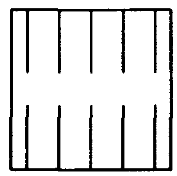

Figure 4 - Disk-loaded structure with 5 periods shorted at the ends.

The resonant frequencies ω_n are noted at points of maximum signal transmission. The resonant frequencies correspond to multiples of half wavelengths along the length of the circuit. If $NL = n\lambda_g/2$ and $\beta = 2\pi/\lambda_g$ then

$$\beta L = \frac{n}{N}\pi \tag{18}$$

where n is an integer from 0 to N. For a forward fundamental structure, the highest resonant frequency corresponds to the largest β and additional periods added to the structure result in lower resonant frequencies.

The interaction impedance is obtained by measuring the dispersion characteristics with a perturbing dielectric rod having a small radius introduced along the symmetry axis of the structure. The total time average stored field energy in the circuit of length NL is $\overline{U} = (1/2)NL\overline{W}$ where $\overline{W}$ is the time average stored energy per period.

When a perturbing object with polarization $\vec{P}$ and magnetization $\vec{M}$ is introduced along the axial circuit length, the resulting relative change in resonant frequency f is

$$\frac{\delta f}{f} = -\frac{\int_v \left(\vec{P}^* \cdot \vec{E} + \vec{M}^* \cdot \vec{B}\right) \mathrm{d}v}{2NL\overline{W}} \tag{19}$$

where the integration is performed over the volume v of the perturbing object. For a thin dielectric rod of radius r positioned along the z-axis of the circuit, only the z-component of the electric field should be significantly altered.

In the case of a dielectric rod of relative permittivity ϵ_r, the polarization $\vec{P} = (\epsilon_r - 1)\epsilon_0\vec{E}$ and the magnetization $\vec{M} = 0$. Therefore the relative change in frequency due to the insertion of the dielectric rod becomes

$$\frac{\delta f}{f} = -\frac{(\epsilon_r - 1)\epsilon_0}{2NL\overline{W}}\pi r^2 \int_0^{NL} E_z^{\,2}\,\mathrm{d}z = -\frac{1}{2}\pi r^2(\epsilon_r - 1)\epsilon_0\frac{\overline{E_z^{\,2}}}{\overline{W}} \tag{20}$$

where

$$\overline{E_z^{\,2}} = \frac{1}{NL}\int_0^{NL} E_z^{\,2}\,\mathrm{d}z. \tag{21}$$

The average interaction impedance is then found to be

$$\overline{Z} = -\frac{1}{\beta^2 v_g} \frac{\delta f}{f} \frac{1}{(\epsilon_r - 1)\epsilon_0 \pi r^2}. \tag{22}$$

The group velocities were calculated from the derivative of the equation describing the dispersion characteristics according to the following relation:

$$v_g = 2L \frac{\mathrm{d}f}{\mathrm{d}(\beta L/\pi)}. \tag{23}$$

Uniform brass disks with center holes of 0.09375 inch radius and thicknesses of 0.003, 0.005, 0.008, 0.010, and 0.020 inch were machined. Copper spacers, 0.080 inch thick with center holes of 0.3 inch radius to act as the cylindrical wave guide, were then stacked with the brass disks to form disk-loaded cylindrical wave guide structures. The slot length between disks was 0.16 inch. The lengths of the structures measured were from one to ten periods. The resonant frequencies of the structures were measured using an HP8510C automatic network analyzer (Figures 5, 6, 7, 8, and 9). The signal coupling was achieved by using coaxial probes on both ends of the structure so as to launch a TM_{01} mode.

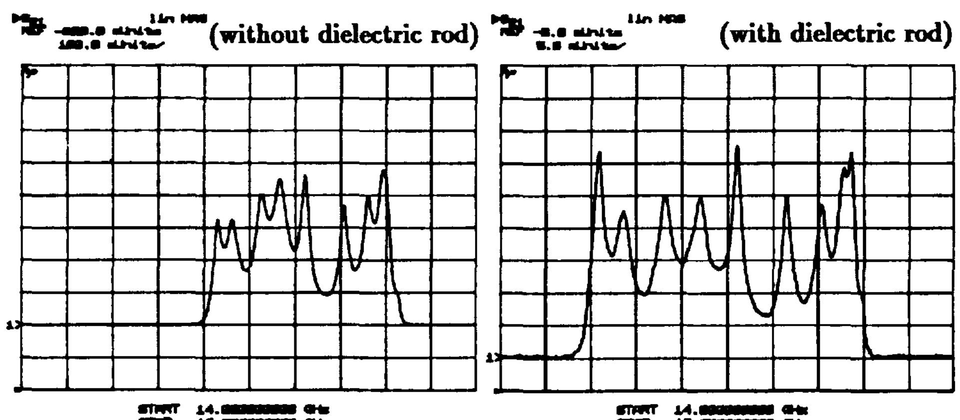

Figure 5 - Transmission measurement using 0.003 inch thick disks.

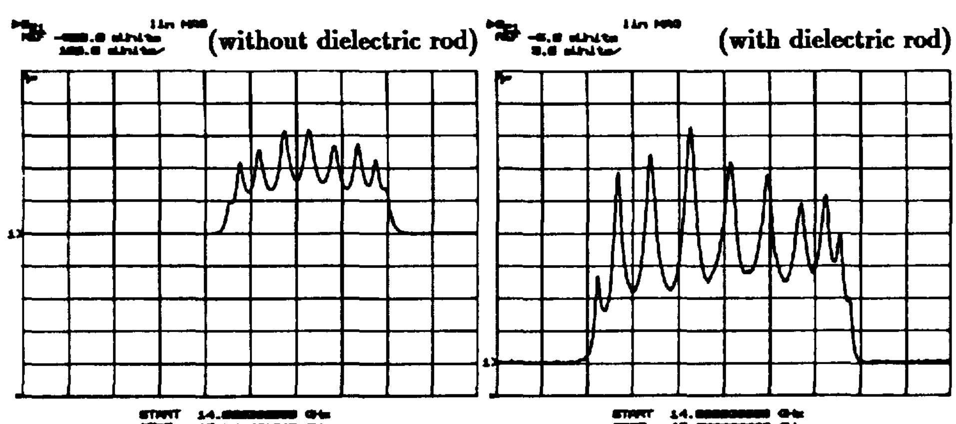

Figure 6 - Transmission measurement using 0.005 inch thick disks.

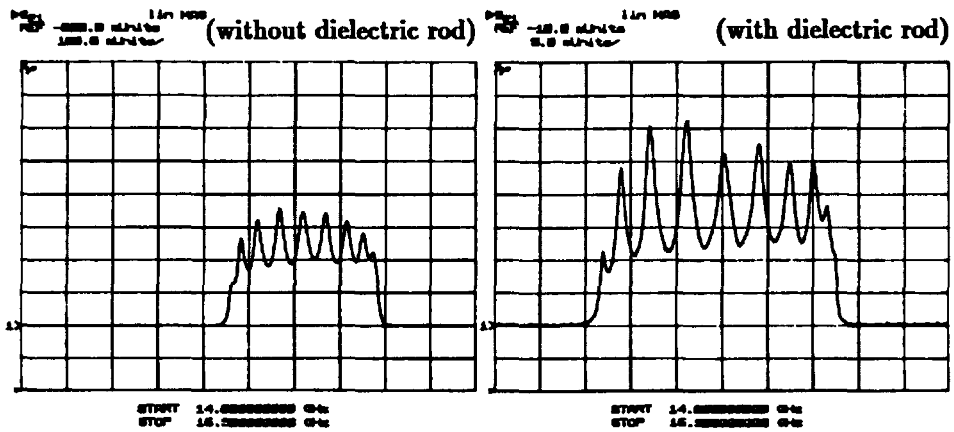

Figure 7 - Transmission measurement using 0.008 inch thick disks.

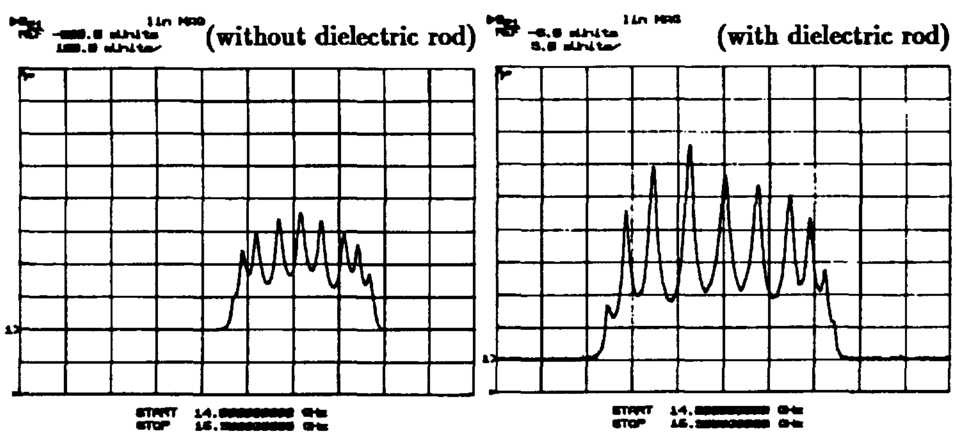

Figure 8 - Transmission measurement using 0.010 inch thick disks.

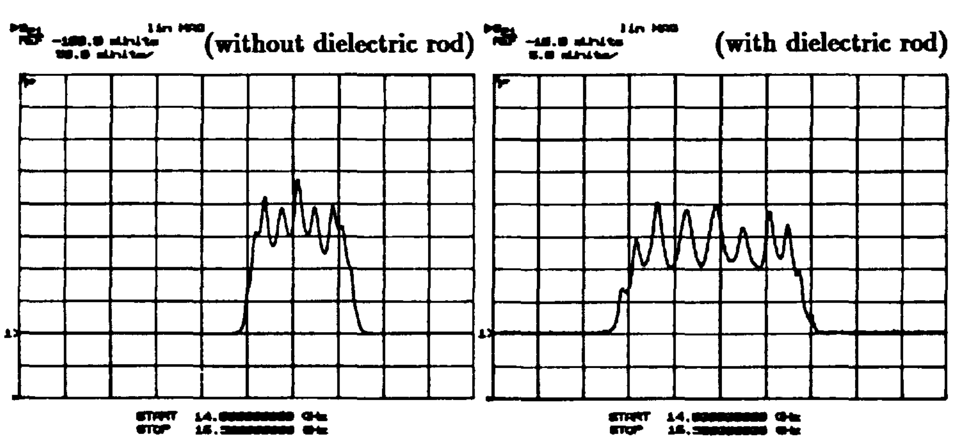

Figure 9 - Transmission measurement using 0.020 inch thick disks.

The dispersion characteristics were fitted to a cubic equation in $\beta L/\pi$ (Figure 10).

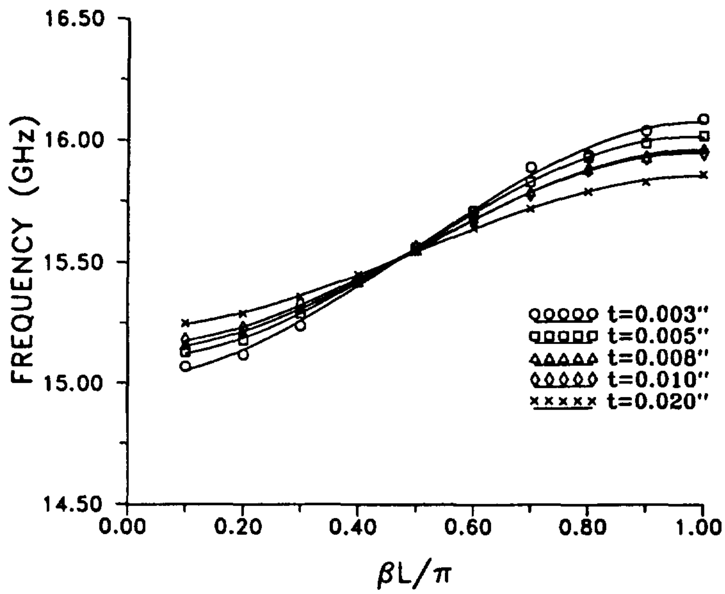

Figure 10 - Experimental dispersion characteristics of the disk-loaded structures.

To determine the interaction impedance, an alumina ($\epsilon_r \approx 9.2$) rod of radius $r = 0.0155$ inches was placed along the length of the symmetry axis of the structure and the resonant frequencies were again measured and identified with the corresponding β as given by equation (18).

From the resulting shifts in frequencies and calculated group velocities, the average interaction impedances were found from equation (22) and fitted using a cubic spline (Figure 11).

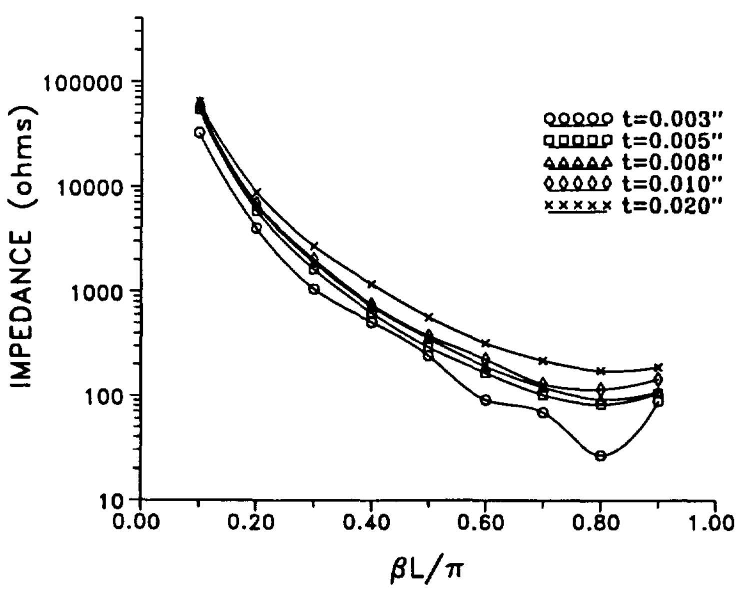

Figure 11 - Experimental interaction impedances of the disk-loaded structures.

COMPUTATIONAL EVALUATION

Numerical calculations of the resonant frequencies for the 0.010 inch thick disk-loaded wave guides were carried out using the two computer codes ARGUS [8] and SOS [9]. These dispersion characteristics and average interaction impedances were obtained using the same equations that were used for calculation of the experimental results.

ARGUS is a three dimensional, time dependent or frequency domain electrodynamic and plasma simulation code. It is a library of programs built around a unifying framework which utilizes the finite difference cell. It contains iterative and non-iterative (direct) Poisson solvers for electrostatic problems and also contains an explicit leapfrog Maxwell solver for electromagnetic problems. The code can simulate plasmas via an implementation of the particle-in-cell (PIC) method.

The Self-Optimized-Sector code, SOS, is another three-dimensional, finite difference code for simulating electromagnetic processes which involve space charge and electromagnetic fields and their interactions. A full set of Maxwell's time-dependent equations are solved to obtain electromagnetic fields and the Lorentz force equation is solved to obtain relative particle trajectories, providing current and charge densities.

A ten period disk-loaded wave guide structure was modeled using the ARGUS computer code on the NASA Lewis CRAY Y-MP (Figure 12).

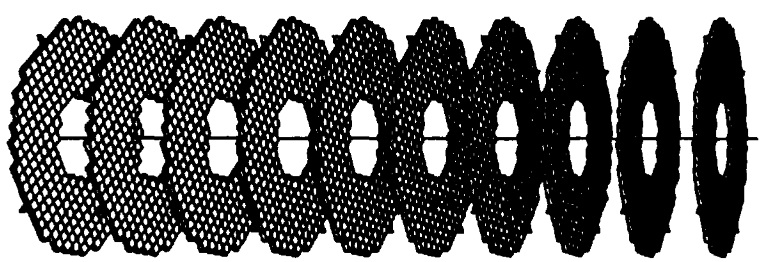

Figure 12 - Ten disk ARGUS mesh showing the simulated alumina rod through the axis.

The structure was modeled in a Cartesian coordinate system since the frequency domain algorithm in ARGUS can only be used with this geometry. Therefore the cylindrical guide and holes in the disks can only be approximated by rectangular elements. The ARGUS code, however, will allow the simulated dielectric rod to have a circular cross section with the same radius as that of the actual dielectric rod used for the frequency measurements.

A uniform mesh with a reasonable grid size was chosen to simplify the input file for the code. Resonant frequencies were found for the structure with and without a simulated alumina dielectric rod ($r = 0.0155$ inches, $\epsilon_r = 9.2$) along the symmetry axis. To obtain more resonances, the number of periods in the structure must be increased. Since the lowest resonance frequencies are the frequencies found first, lengthening the structure insures that these are the axial resonances.

The SOS code was run on the NASA Lewis Scientific VAX cluster using one and two periods of the structures. For purposes of comparison to ARGUS, a Cartesian geometry was chosen to model the disk-loaded cylindrical structure even though the frequency domain algorithm in SOS will work with cylindrical geometry. The SOS code used with a Cartesian coordinate system does not allow the modeling of the dielectric rod with a circular cross-section, therefore a dielectric rod with a square cross-section was used. A uniform mesh was chosen so that the wave guide diameter was an integral number of elements such that the area of an element was approximately that of the dielectric rod.

A simulated dielectric rod having a rectangular cross section 15% larger than the actual rod was used. The difference in the dielectric rod cross-sectional area would be reflected in a difference in frequency shift and only have a negligible effect on the values of interaction impedances obtained. By using a combination of conductive walls and mirror symmetry boundary conditions at the ends of the structures the 0, $\pi/4$, $\pi/2$, $3\pi/4$, and π points of the dispersion characteristics were found.

COMPARISON OF RESULTS

Results obtained through theoretical calculations are compared with those calculated from the measured values of resonant frequencies of unperturbed and perturbed 0.010" thick disk-loaded structure and those of the two computer codes. The dispersion characteristics are shown for the disk-loaded circular wave guide structure obtained using theoretical, experimental, and computational techniques (Figure 13). Each set of frequency data was again fitted to a cubic equation in $\beta L/\pi$.

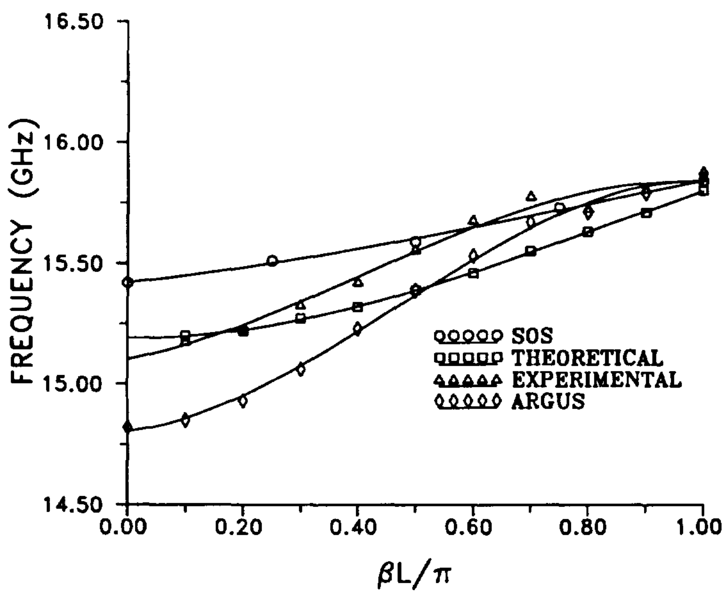

Figure 13 - Dispersion characteristics of the 0.010 inch thick disk-loaded structures.

Resonance frequencies found using SOS are close to those obtained experimentally especially near $\beta L/\pi = 0.5$. The maximum deviation was only 1.63%. Frequency values generated by ARGUS were typically lower deviating from 2.3% at $\beta L/\pi = 0$ to 0.75% at $\beta L/\pi = 1$. Frequency values predicted by the theoretical approach agree very well at lower frequencies with measured results but tend to be lower at the higher frequencies with as much as a 1.7% deviation. They seem to agree very closely to the ARGUS values near $\beta L/\pi = 0.5$.

The average interaction impedances are also shown for the disk-loaded circular wave guide structure obtained using theoretical, experimental, and computational techniques (Figure 14). The four data sets were fitted to a cubic spline in $\beta L/\pi$.

The values of interaction impedance obtained through the use of the SOS code closely agree with the theoretical values. This may be due to the similarity in form of the respective dispersion characteristics. Likewise, the values of interaction impedance using data from the ARGUS code agree extremely well with the experimental results. In this case the form of the respective dispersion characteristics are also similar. The similarity in characteristics result in group velocities and hence interaction impedances that are nearly the same.

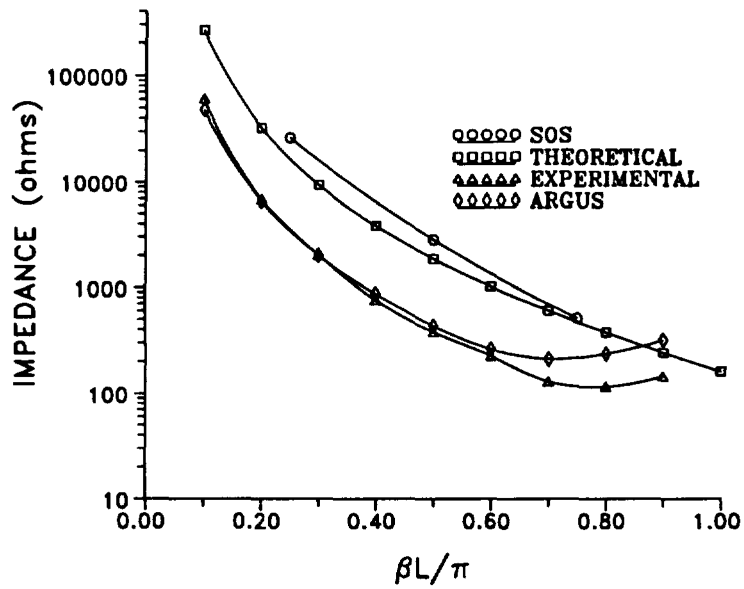

Figure 14 - Interaction impedance characteristics of the 0.010 inch thick disk-loaded structures.

Due to the simplifications in the derivation of equations (14) and (22) the values of interaction impedance in all cases are very high and serve only as an index of comparison between the different approaches. Also the method of calculating the group velocities lends uncertainty to the values of interaction impedances. In general, the behavior of impedance is consistently similar in all the four cases. The minor deviations of the computer values from the experimental values can be attributed to the computational approaches used in ARGUS and SOS.

The theoretical results incorporated several assumptions in the theory. The fields in the slot are assumed to be radial TEM and for $\lambda_g \gg t + s$ this assumption is justified. Space harmonics which would introduce a $\sin(\beta_n s)/(\beta_n s)$ factor for the nth harmonic into the analysis have been ignored. Admittance matching across the slot at $\rho = a$ is done in the average sense using the space factor η to account for the electric field being zero on the disk edge. Actually a point by point match cannot be done and so inaccuracies will inevitably occur in these results.

The design of structures using the theoretical approach in association with an appropriate computer code can be greatly facilitated and can provide an insight in the performance of a structure prior to its being fabricated and tested. The disk-loaded structure is a good candidate for use in miniature traveling-wave tubes. With the proper choice of dimensions a disk-loaded device with good interaction impedance operating at a nominal beam voltage may be possible.

CIRCUIT MODELING

The disk-loaded structure lends itself readily to a circuit modeling approach. A way to describe one period of the structure is to use a series of four cylindrical wave guides, two cylinders with the radii equal to the radius a of the disk hole and having a length equal to the half the thickness $t/2$ of the disk between two cylinders each having radii equal to the radius b of the disk-loaded structure and having lengths equal to half the gap spacing $s/2$.

Harrington [10] provides equivalent circuits of a cylinder for both TE and TM modes. In slow-wave structures for traveling-wave tube applications we are interested in the TM modes only since these provide a non-zero axial electric field which can couple to an electron beam.

For the lowest order cylindrical TM mode, a TM_{01} mode, the circuit equivalent values per unit length for impedance Z and admittance Y for a cylinder of radius r are given by

$$Z = jX = j\omega\mu - j\,\frac{k_c^{\,2}}{\omega\epsilon} \tag{24}$$

and

$$Y = jB = j\omega\epsilon \tag{25}$$

where $k_c = 2.405/r$ is the cutoff wave number.

The values for the characteristic propagation constant γ and characteristic impedance Z_0 for the TM_{01} equivalent circuit can also be found using general transmission theory.
The characteristic propagation constant and impedance are given as

$$\gamma_{TM} = \sqrt{ZY} = j\beta = jk\sqrt{1-(k_c/k)^2} \tag{26}$$

and

$$Z_{0TM} = \sqrt{Z/Y} = \frac{\beta}{\omega\epsilon} = \sqrt{\mu/\epsilon}\sqrt{1-(k_c/k)^2} \tag{27}$$

where $k = \omega\sqrt{\mu\epsilon}$ is the wave number.

Circuit elements having values corresponding to the 0.010 inch thick disk-loaded wave guide structure were used to model cylinder sections. The circuit modeling was performed using Touchstone. Touchstone is a linear, frequency domain circuit program which can perform simulation and optimization of an electrical circuit. The program resides on a SUN SPARC workstation at NASA Lewis Research Center.

Circuit elements were also needed to account for discontinuities at the ends and between dissimilar cylinders. Circuit elements with a variable series LC impedance and a variable parallel LC admittance were chosen to model the discontinuities. The entire circuit was kept symmetric with respect to the center of the structure (Figure 15).

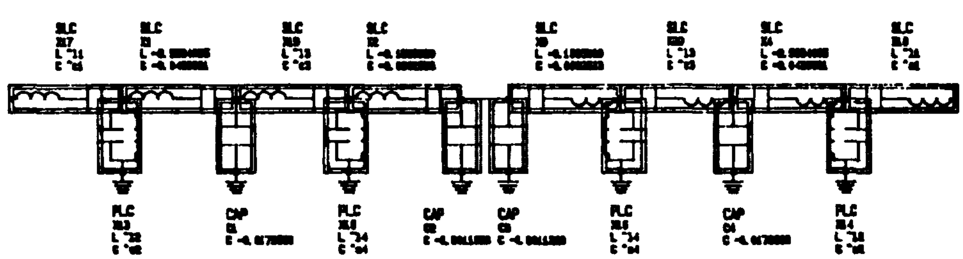

Figure 15 - Touchstone model of the 0.010 inch disk-loaded circular wave guide.

The values for the inductances and capacitances for the variable elements were found by optimizing the circuit such that maximum transmission occurred at the $\beta L = 0$ and $\beta L = \pi$ experimentally measured cutoff frequencies of the structure (Figure 16). The transmission is lossless at these frequencies since no resistors were introduced into the circuit.

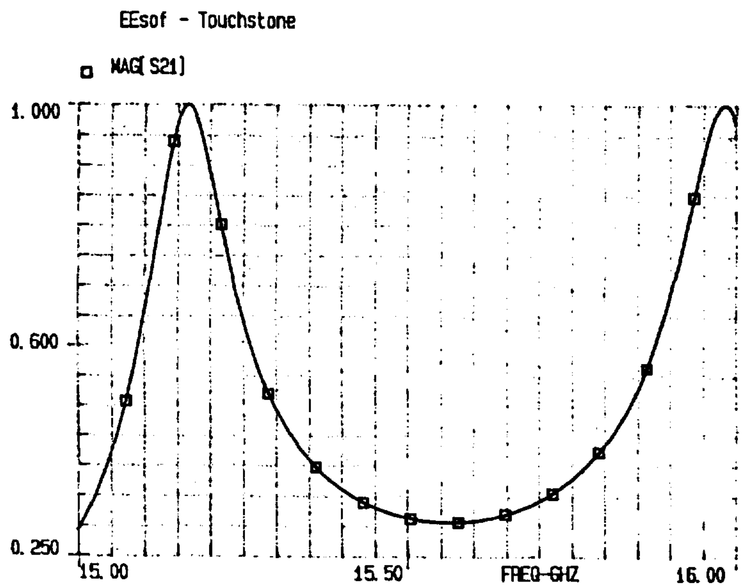

Figure 16 - Transmission simulation using Touchstone.

This effort shows that it is possible to model a period of a given slow-wave structure using the $\beta L = 0$ and $\beta L = \pi$ known resonance frequencies. It is a matter of adding resistances to account for the losses observed.

ACKNOWLEDGEMENT

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APPENDIX A

Mathcad program to calculate frequency and interaction impedance for a disk-loaded circular wave guide TM_{01} mode given the inner radius, outer radius, slot spacing, disk thickness, and axial wavenumber.

inner radius	$a := .09375 \cdot .0254$	**speed of light**	$c := 2.99792 \cdot 10^{8}$	**TE 11 cutoff**
outer radius	$b := .3 \cdot .0254$	**electronic charge**	$q := 1.60218 \cdot 10^{-19}$	$\frac{1.841}{2 \cdot \pi \cdot b} \cdot c \cdot 10^{-9} = 11.527602$
slot spacing	$s := 2 \cdot .08 \cdot .0254$	**electronic mass**	$m := 9.1095 \cdot 10^{-31}$	**TM 01 cutoff**
disk thickness	$t := .020 \cdot .0254$	**permeability**	$\mu := 4 \cdot \pi \cdot 10^{-7}$	$\frac{2.405}{2 \cdot \pi \cdot b} \cdot c \cdot 10^{-9} = 15.059144$
period	$d := s + t$	**permittivity**	$\varepsilon := 8.854215 \cdot 10^{-12}$	**TE 21 cutoff**
spatial factor	$\eta := \frac{t}{d}$	**axial wavenumber**	$\beta := (.8) \cdot \frac{\pi}{d}$	$\frac{3.054}{2 \cdot \pi \cdot b} \cdot c \cdot 10^{-9} = 19.122921$

Define Bessel function combinations

$Z00(k,a) := J0(k \cdot a) \cdot Y0(k \cdot b) - Y0(k \cdot a) \cdot J0(k \cdot b)$ $Z01(k,a) := J0(k \cdot a) \cdot Y1(k \cdot b) - Y0(k \cdot a) \cdot J1(k \cdot b)$

$Z10(k,a) := J1(k \cdot a) \cdot Y0(k \cdot b) - Y1(k \cdot a) \cdot J0(k \cdot b)$ $Z11(k,a) := J1(k \cdot a) \cdot Y1(k \cdot b) - Y1(k \cdot a) \cdot J1(k \cdot b)$

Define Bessel function combination derivatives

$Z00'(k,a) := -Z10(k,a) - \frac{b}{a} \cdot Z01(k,a)$ $Z10'(k,a) := Z00(k,a) - \frac{1}{k \cdot a} \cdot Z10(k,a) - \frac{b}{a} \cdot Z11(k,a)$

Define α and the derivative α'

$\alpha(k,a) := \frac{1}{k \cdot a} \cdot \frac{Z10(k,a)}{Z00(k,a)}$ $\alpha'(k,a) := \alpha(k,a) \cdot \left(\frac{Z10'(k,a)}{Z10(k,a)} - \frac{Z00'(k,a)}{Z00(k,a)} - \frac{1}{k \cdot a} \right)$

Define modified Bessel functions

$j := 0 .. 15$ $I0(x) := \sum_{j} \frac{\left(\frac{x}{2}\right)^{2 \cdot j}}{(j!)^2}$ $I1(x) := \sum_{j} \frac{\left(\frac{x}{2}\right)^{2 \cdot j + 1}}{j! \cdot (j+1)!}$ $I2(x) := I0(x) - \frac{2}{x} \cdot I1(x)$

Define modified Bessel function derivatives

$I0'(x) := I1(x)$ $I1'(x) := I0(x) - \frac{I1(x)}{x}$

Define ϕ and the derivative ϕ'

$\phi(\tau,a) := \frac{1}{\tau \cdot a} \cdot \frac{I1(\tau \cdot a)}{I0(\tau \cdot a)}$ $\phi'(\tau,a) := \phi(\tau,a) \cdot \left(\frac{I1'(\tau \cdot a)}{I1(\tau \cdot a)} - \frac{I0'(\tau \cdot a)}{I0(\tau \cdot a)} - \frac{1}{\tau \cdot a} \right)$

Solve for *k* using $\phi = \alpha$

$k := 300$ GIVEN $\phi\left(\sqrt{\beta^2 - k^2}, a\right) = \frac{\alpha(k,a)}{1 - \eta}$ $k := FIND(k)$

Calculate frequency, τ, α, and ϕ

$f := \frac{c \cdot k}{2 \cdot \pi}$ $\tau := \sqrt{\beta^2 - k^2}$ $\phi(\tau,a) = 0.441851$

$f \cdot 10^{-9} = 15.693878$ $\tau = 440.44614$ $\alpha(k,a) = 0.392757$

Calculate the phase velocity

$$v := c\cdot\frac{k}{\beta} \qquad \frac{v}{c} = 0.598352$$

Calculate the beam voltage

$$V := \left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right]\cdot\frac{m}{q}\cdot c^2 \qquad V = 126767.243147$$

Calculate the group velocity

$$d\beta dk := \frac{k\cdot a}{\beta\cdot a} + \frac{\tau\cdot a}{\beta\cdot a}\cdot\frac{\alpha'(k,a)}{\phi'(\tau,a)}\cdot\frac{1}{1-\eta}$$

$$vg := \frac{c}{d\beta dk} \qquad \frac{vg}{c} = 0.023221$$

Calculate the power

$$P := \frac{\pi}{2}\cdot\frac{\beta\cdot k}{\tau\cdot\tau}\cdot a^2\cdot\sqrt{\frac{\varepsilon}{\mu}}\cdot(I1(\tau\cdot a)\cdot I1(\tau\cdot a) - I0(\tau\cdot a)\cdot I2(\tau\cdot a))$$

$$P = 3.635789\cdot 10^{-9}$$

Calculate the interaction impedance

dielectric rod radius $\quad r := 0.0155\cdot .0254 \quad Ez2 := I0(\tau\cdot r)\cdot I0(\tau\cdot r) - I1(\tau\cdot r)\cdot I1(\tau\cdot r) \quad Z := \frac{Ez2}{2\cdot\beta\cdot\beta\cdot P} \quad Z = 458.531069$

Calculate the matching impedance

$$Zi := c\cdot\frac{\beta}{k}\cdot 200\cdot\frac{I0(\tau\cdot a) - 1}{\tau\cdot a\cdot I1(\tau\cdot a)} \qquad \frac{1}{Zi}\cdot 9\cdot 10^{11} = 19.188538$$

Calculate the lower cutoff frequency due to the outer radius *b*

$k := 300 \qquad$ GIVEN $\qquad J0(k\cdot b) = 0 \qquad klow := Find(k) \qquad flow := klow\cdot 10^{-9}\cdot\frac{c}{2\cdot\pi} \qquad flow = 15.058052$

Calculate the upper cutoff frequency due to the inner radius *a*

GIVEN $\quad J1(k\cdot a)\cdot Y0(k\cdot b) - J0(k\cdot b)\cdot Y1(k\cdot a) = 0 \quad khigh := Find(k) \quad fhigha := khigh\cdot 10^{-9}\cdot\frac{c}{2\cdot\pi} \quad fhigha = 17.65171$

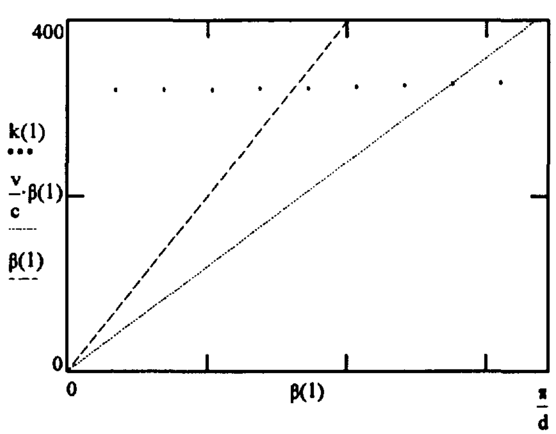
Calculate the upper cutoff frequency due to the period *d*

$$fhighd := \frac{c}{2\cdot d}\cdot 10^{-9} \qquad fhighd = 32.785652$$

Solve for the dispersion characteristics

$$n := 10 \quad l := 0..n \quad \beta(l) := \frac{l}{n}\cdot\frac{\pi}{d} \quad k(l) := root\left[\alpha(Re(k),a) - (1-\eta)\cdot\phi\left(\sqrt{\beta(l)^2 - Re(k)^2},a\right),k\right] \quad f(l) := \frac{c\cdot k(l)}{2\cdot\pi}$$

Plot of the dispersion characteristics



l	$f(l)\cdot 10^{-9}$
0	15.306505
1	15.314177
2	15.336804
3	15.373274
4	15.421881
5	15.480517
6	15.546883
7	15.618684
8	15.693789
9	15.770333
10	15.846771

APPENDIX B

Mathcad program to calculate frequency and interaction impedance for a disk-loaded circular wave guide TM_{01} mode given the inner radius, outer radius, slot spacing, disk thickness, voltage, and current.

inner radius	$a := .09375 \cdot .0254$	**speed of light**	$c := 2.99792 \cdot 10^{8}$	**TE 11 cutoff** $\frac{1.841}{2 \cdot \pi \cdot b} \cdot c \cdot 10^{-9} = 11.527602$
outer radius	$b := .3 \cdot .0254$	**electronic charge**	$q := 1.60218 \cdot 10^{-19}$	**TM 01 cutoff** $\frac{2.405}{2 \cdot \pi \cdot b} \cdot c \cdot 10^{-9} = 15.059144$
slot spacing	$s := 2 \cdot .08 \cdot .0254$	**electronic mass**	$m := 9.1095 \cdot 10^{-31}$	**TE 21 cutoff** $\frac{3.054}{2 \cdot \pi \cdot b} \cdot c \cdot 10^{-9} = 19.122921$
disk thickness	$t := .010 \cdot .0254$	**permeability**	$\mu := 4 \cdot \pi \cdot 10^{-7}$	
period	$d := s + t$	**permittivity**	$\varepsilon := 8.854215 \cdot 10^{-12}$	
spatial factor	$\eta := \frac{t}{d}$	**beam voltage**	$V := 60000$	
		beam current	$I := .001$	

Calculate the phase velocity

$$v := \sqrt{\frac{\left(\frac{2 \cdot q \cdot V}{m \cdot c}\right)^2}{2} \cdot \left[\sqrt{1 + \left(\frac{m \cdot c^2}{q \cdot V}\right)^2} - 1\right]} \qquad \frac{v}{c} = 0.457026$$

Define Bessel function combinations

$$Z00(k,a) := J0(k \cdot a) \cdot Y0(k \cdot b) - Y0(k \cdot a) \cdot J0(k \cdot b) \qquad Z01(k,a) := J0(k \cdot a) \cdot Y1(k \cdot b) - Y0(k \cdot a) \cdot J1(k \cdot b)$$

$$Z10(k,a) := J1(k \cdot a) \cdot Y0(k \cdot b) - Y1(k \cdot a) \cdot J0(k \cdot b) \qquad Z11(k,a) := J1(k \cdot a) \cdot Y1(k \cdot b) - Y1(k \cdot a) \cdot J1(k \cdot b)$$

Define Bessel function combination derivatives

$$Z00'(k,a) := -Z10(k,a) - \frac{b}{a} \cdot Z01(k,a) \qquad Z10'(k,a) := Z00(k,a) - \frac{1}{k \cdot a} \cdot Z10(k,a) - \frac{b}{a} \cdot Z11(k,a)$$

Define α and the derivative α'

$$\alpha(k,a) := \frac{1}{k \cdot a} \cdot \frac{Z10(k,a)}{Z00(k,a)} \qquad \alpha'(k,a) := \alpha(k,a) \cdot \left(\frac{Z10'(k,a)}{Z10(k,a)} - \frac{Z00'(k,a)}{Z00(k,a)} - \frac{1}{k \cdot a}\right)$$

Define modified Bessel functions

$$j := 0..15 \qquad I0(x) := \sum_j \frac{\left(\frac{x}{2}\right)^{2 \cdot j}}{(j!)^2} \qquad I1(x) := \sum_j \frac{\left(\frac{x}{2}\right)^{2 \cdot j + 1}}{j! \cdot (j+1)!} \qquad I2(x) := I0(x) - \frac{2}{x} \cdot I1(x)$$

Define modified Bessel function derivatives

$$I0'(x) := I1(x) \qquad I1'(x) := I0(x) - \frac{I1(x)}{x}$$

Define ϕ and the derivative ϕ'

$$\phi(\tau,a) := \frac{1}{\tau \cdot a} \cdot \frac{I1(\tau \cdot a)}{I0(\tau \cdot a)} \qquad \phi'(\tau,a) := \phi(\tau,a) \cdot \left(\frac{I1'(\tau \cdot a)}{I1(\tau \cdot a)} - \frac{I0'(\tau \cdot a)}{I0(\tau \cdot a)} - \frac{1}{\tau \cdot a}\right)$$

Solve for *k* using $\phi = \alpha$

$$k := 300 \qquad \text{GIVEN} \qquad \phi\left[k \cdot \sqrt{\left(\frac{c}{v}\right)^2 - 1}, a\right] = \frac{\alpha(k,a)}{1 - \eta} \qquad k := \text{FIND}(k)$$

Calculate frequency, β, τ, α, and φ

$f := \frac{c \cdot k}{2 \cdot \pi}$ $\beta := \frac{c}{v} \cdot k$ $\tau := \sqrt{\beta^2 - k^2}$ $\alpha(k,a) = 0.37084$

$f \cdot 10^{-9} = 15.79377$ $\beta = 724.277576$ $\tau = 644.211229$ $\phi(\tau,a) = 0.394017$

Calculate the group velocity

$d\beta dk := \frac{k \cdot a}{\beta \cdot a} + \frac{\tau \cdot a}{\beta \cdot a} \cdot \frac{\alpha'(k,a)}{\phi'(\tau,a)} \cdot \frac{1}{1 - \eta}$

$vg := \frac{c}{d\beta dk}$ $\frac{vg}{c} = 0.023943$

Calculate the power

$P := \frac{\pi}{2} \cdot \frac{\beta \cdot k}{\tau \cdot \tau} \cdot a^2 \cdot \sqrt{\frac{\varepsilon}{\mu}} \cdot (I1(\tau \cdot a) \cdot I1(\tau \cdot a) - I0(\tau \cdot a) \cdot I2(\tau \cdot a))$

$P = 5.91736 \cdot 10^{-9}$

Calculate the interaction impedance

dielectric rod radius $r := .0155 \cdot .0254$ $Ez2 := I0(\tau \cdot r) \cdot I0(\tau \cdot r) - I1(\tau \cdot r) \cdot I1(\tau \cdot r)$ $Z := \frac{Ez2}{2 \cdot \beta \cdot \beta \cdot P}$ $Z = 163.687697$

Calculate the matching impedance

$Zi := c \cdot \frac{\beta}{k} \cdot 200 \cdot \frac{I0(\tau \cdot a) - 1}{\tau \cdot a \cdot I1(\tau \cdot a)}$ $\frac{1}{Zi} \cdot 9 \cdot 10^{11} = 15.705308$

Calculate the lower cutoff frequency due to the outer radius *b*

$k := 300$ GIVEN $J0(k \cdot b) = 0$ $klow := Find(k)$ $flow := klow \cdot 10^{-9} \cdot \frac{c}{2 \cdot \pi}$ $flow = 15.058052$

Calculate the upper cutoff frequency due to the inner radius *a*

GIVEN $J1(k \cdot a) \cdot Y0(k \cdot b) - J0(k \cdot b) \cdot Y1(k \cdot a) = 0$ $khigh := Find(k)$ $fhigha := khigh \cdot 10^{-9} \cdot \frac{c}{2 \cdot \pi}$ $fhigha = 17.65171$

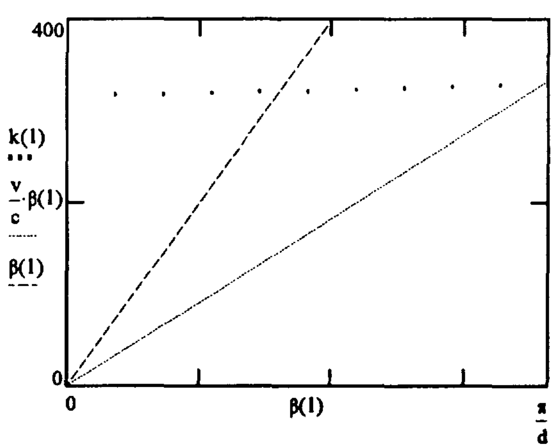
Calculate the upper cutoff frequency due to the period *d*

$fhighd := \frac{c}{2 \cdot d} \cdot 10^{-9}$ $fhighd = 34.71422$

Solve for the dispersion characteristics

$n := 10$ $l := 0..n$ $\beta(l) := \frac{l}{n} \cdot \frac{\pi}{d}$ $k(l) := root\left[\alpha(Re(k),a) - (1 - \eta) \cdot \phi\left(\sqrt{\beta(l)^2 - Re(k)^2}, a\right), k\right]$ $f(l) := \frac{c \cdot k(l)}{2 \cdot \pi}$

Plot of the dispersion characteristics



l	$f(l) \cdot 10^{-9}$
0	15.185202
1	15.19403
2	15.220011
3	15.261706
4	15.321726
5	15.38878
6	15.464114
7	15.544958
8	15.628804
9	15.713518
10	15.797392

APPENDIX C

Mathcad program to calculate the inner radius and interaction impedance for a disk-loaded circular wave guide TM_{01} mode given the frequency, outer radius, period, disk thickness, voltage, and current.

outer radius $b := .006625$

beam voltage $V := 5000$

beam current $I := .001$

frequency $f := 30\cdot 10^{9}$

disk thickness $t := .00006858$

speed of light $c := 2.99792\cdot 10^{8}$

electronic charge $q := 1.60218\cdot 10^{-19}$

electronic mass $m := 9.1095\cdot 10^{-31}$

permeability $\mu := 4\cdot\pi\cdot 10^{-7}$

permittivity $\varepsilon := 8.854215\cdot 10^{-12}$

TE 11 cutoff
$\frac{1.841}{2\cdot\pi\cdot b}\cdot c\cdot 10^{-9} = 13.258918$

TM 01 cutoff
$\frac{2.405}{2\cdot\pi\cdot b}\cdot c\cdot 10^{-9} = 17.320857$

TE 21 cutoff
$\frac{3.054}{2\cdot\pi\cdot b}\cdot c\cdot 10^{-9} = 21.994967$

minimum outer radius $\frac{2.405}{2\cdot\pi\cdot f}\cdot c = 0.003825$ $\qquad k := \frac{2\cdot\pi\cdot f}{c}$

Calculate the phase velocity, *k*, β, and τ

$$v := \sqrt{\frac{\left(\frac{2\cdot q\cdot V}{m\cdot c}\right)^2}{2}\cdot\left[\sqrt{1+\left(\frac{m\cdot c^2}{q\cdot V}\right)^2}-1\right]} \qquad \frac{v}{c} = 0.139208 \qquad \beta := \frac{c}{v}\cdot k \qquad \tau := \sqrt{\beta^2 - k^2}$$

Choose the period *d*

maximum period $\frac{\pi}{\beta} = 0.000696$ $\qquad$ **period** $d := .4\cdot\frac{\pi}{\beta}$ $\quad d = 0.000278$ $\qquad$ **spatial factor** $\eta := \frac{t}{d}$ $\quad \eta = 0.246493$

Define Bessel function combinations

$$Z00(k,a) := J0(k\cdot a)\cdot Y0(k\cdot b) - Y0(k\cdot a)\cdot J0(k\cdot b) \qquad Z01(k,a) := J0(k\cdot a)\cdot Y1(k\cdot b) - Y0(k\cdot a)\cdot J1(k\cdot b)$$

$$Z10(k,a) := J1(k\cdot a)\cdot Y0(k\cdot b) - Y1(k\cdot a)\cdot J0(k\cdot b) \qquad Z11(k,a) := J1(k\cdot a)\cdot Y1(k\cdot b) - Y1(k\cdot a)\cdot J1(k\cdot b)$$

Define Bessel function combination derivatives

$$Z00'(k,a) := -Z10(k,a) - \frac{b}{a}\cdot Z01(k,a) \qquad Z10'(k,a) := Z00(k,a) - \frac{1}{k\cdot a}\cdot Z10(k,a) - \frac{b}{a}\cdot Z11(k,a)$$

Define α and the derivative α'

$$\alpha(k,a) := \frac{1}{k\cdot a}\cdot\frac{Z10(k,a)}{Z00(k,a)} \qquad \alpha'(k,a) := \alpha(k,a)\cdot\left(\frac{Z10'(k,a)}{Z10(k,a)} - \frac{Z00'(k,a)}{Z00(k,a)} - \frac{1}{k\cdot a}\right)$$

Define modified Bessel functions

$$j := 0..15 \qquad I0(x) := \sum_j \frac{x^{2\cdot j}}{j!\cdot j!\cdot 2^{2\cdot j}} \qquad I1(x) := \sum_j \frac{x^{2\cdot j+1}}{j!\cdot(j+1)!\cdot 2^{2\cdot j+1}} \qquad I2(x) := I0(x) - \frac{2}{x}\cdot I1(x)$$

Define modified Bessel function derivatives

$$I0'(x) := I1(x) \qquad I1'(x) := I0(x) - \frac{I1(x)}{x}$$

Define ϕ and the derivative ϕ'

$$\phi(\tau,a) := \frac{1}{\tau\cdot a}\cdot\frac{I1(\tau\cdot a)}{I0(\tau\cdot a)} \qquad \phi'(\tau,a) := \phi(\tau,a)\cdot\left(\frac{I1'(\tau\cdot a)}{I1(\tau\cdot a)} - \frac{I0'(\tau\cdot a)}{I0(\tau\cdot a)} - \frac{1}{\tau\cdot a}\right)$$

Solve for the inner radius *a*

$a := .5\cdot b \quad \text{GIVEN} \quad \phi(\tau,a) = \dfrac{\alpha(k,a)}{1-\eta} \quad a := \text{FIND}(a) \quad a = 0.004015$

Calculate ϕ and α

$\phi(\tau,a) = 0.054157 \qquad \alpha(k,a) = 0.040808$

Calculate the group velocity

$$d\beta dk := \frac{k\cdot a}{\beta\cdot a} + \frac{\tau\cdot a}{\beta\cdot a}\cdot\frac{\alpha'(k,a)}{\phi'(\tau,a)}\cdot\frac{1}{1-\eta}$$

$$vg := \frac{c}{d\beta dk} \qquad \frac{vg}{c} = 0.007476$$

Calculate the power

$$P := \frac{\pi}{2}\cdot\frac{\beta}{\tau}\cdot\frac{k}{\tau}\cdot a^2\cdot\sqrt{\frac{\varepsilon}{\mu}}\cdot\left(I1(\tau\cdot a)\cdot I1(\tau\cdot a) - I0(\tau\cdot a)\cdot I2(\tau\cdot a)\right)$$

$P = 18288.69785$

Calculate the interaction impedance

dielectric rod radius $r := .0155\cdot .0254 \quad Ez2 := I0(\tau\cdot r)\cdot I0(\tau\cdot r) - I1(\tau\cdot r)\cdot I1(\tau\cdot r) \quad Z := \dfrac{Ez2}{2\cdot\beta\cdot\beta\cdot P} \quad Z = 2.881309\cdot 10^{-12}$

Calculate the matching impedance

$$Zi := c\cdot\frac{\beta}{k}\cdot 200\cdot\frac{I0(\tau\cdot a) - 1}{\tau\cdot a\cdot I1(\tau\cdot a)} \qquad \frac{1}{Zi}\cdot 9\cdot 10^{11} = 36.490534$$

Calculate the Pierce parameter

$$C := \left(\frac{I\cdot Z}{4\cdot V}\right)^{.33333} \qquad C = 5.243035\cdot 10^{-7}$$

Calculate the rf power output

$2\cdot C\cdot V\cdot I = 0.000005$

Calculate the lower cutoff frequency due to the outer radius *b*

$k := 300 \quad \text{GIVEN} \quad J0(k\cdot b) = 0 \quad klow := \text{Find}(k) \quad flow := klow\cdot 10^{-9}\cdot\dfrac{c}{2\cdot\pi} \quad flow = 17.3196$

Calculate the upper cutoff frequency due to the inner radius *a*

$\text{GIVEN} \quad J1(k\cdot a)\cdot Y0(k\cdot b) - J0(k\cdot b)\cdot Y1(k\cdot a) = 0 \quad khigh := \text{Find}(k) \quad fhigha := khigh\cdot 10^{-9}\cdot\dfrac{c}{2\cdot\pi} \quad fhigha = 31.665994$

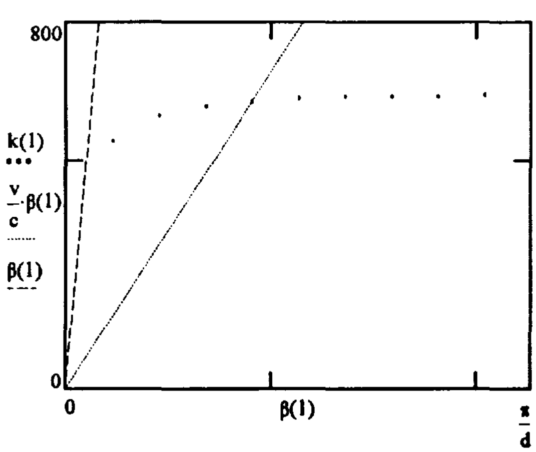
Calculate the upper cutoff frequency due to the period *d*

$$fhighd := \frac{c}{2\cdot d}\cdot 10^{-9} \qquad fhighd = 538.762802$$

Solve for the disperion characteristics

$n := 10 \quad l := 0..n \quad \beta(l) := \dfrac{l}{n}\cdot\dfrac{\pi}{d} \quad k(l) := \text{root}\left[\alpha(\text{Re}(k),a) - (1-\eta)\cdot\phi\left(\sqrt{\beta(l)^2 - \text{Re}(k)^2},a\right),k\right] \quad f(l) := \dfrac{c\cdot k(l)}{2\cdot\pi}$

Plot of the dispersion characteristics



l	$f(l)\cdot 10^{-9}$
0	18.646066
1	25.926408
2	28.514995
3	29.487839
4	29.998627
5	30.300609
6	30.469927
7	30.556855
8	30.601727
9	30.62655
10	30.641434

APPENDIX D

Mathcad program to calculate the interaction impedance for a disk-loaded circular wave guide TM_{01} mode given experimental or computational data.

inner radius	$a := .09375 \cdot .0254$	**speed of light**	$c := 2.99792 \cdot 10^{8}$	**Read frequency data file**	
outer radius	$b := .3 \cdot .0254$	**electronic charge**	$q := 1.60218 \cdot 10^{-19}$	$M := \mathrm{READPRN}(dcwg0010)$	
disk thickness	$t := .003 \cdot .0254$	**electronic mass**	$m := 9.1095 \cdot 10^{-31}$	**Unperturbed frequencies**	
slot spacing	$s := .16 \cdot .0254$	**permeability**	$\mu := 4 \cdot \pi \cdot 10^{-7}$	$f := M^{\langle 0 \rangle} \cdot 10^{9}$	
period	$l := s + t$	**permittivity**	$\varepsilon := 8.854215 \cdot 10^{-12}$	$f := \vec{f}$	
dielectric rod radius	$r := .0155 \cdot .0254$	**dielectric constant**	$\varepsilon r := 9.2$	**Perturbed frequencies**	
				$fd := M^{\langle 1 \rangle} \cdot 10^{9}$	
Frequency shifts	$\delta f(i) := f_i - fd_i$	**number of points**	$N := \mathrm{length}(f)$	$fd := \overrightarrow{fd}$	

Least squares cubic fit in β

$i := 0 .. N - 1 \qquad D := 3 \qquad d := 0 .. D \qquad \beta_i := \frac{i+1}{N} \cdot \frac{\pi}{l} \qquad \beta := \vec{\beta} \qquad X^{\langle d \rangle} := \overrightarrow{\left(\beta^{d}\right)}$

Calculate the coefficients

$\omega := 2 \cdot \pi \cdot f \qquad coeff := \left(X^{T} \cdot X\right)^{-1} \cdot \left(X^{T} \cdot \omega\right) \qquad \Omega(x) := \sum_{d} coeff_d \cdot x^{d} \qquad \Omega'(x) := \sum_{d} coeff_d \cdot d \cdot x^{d-1}$

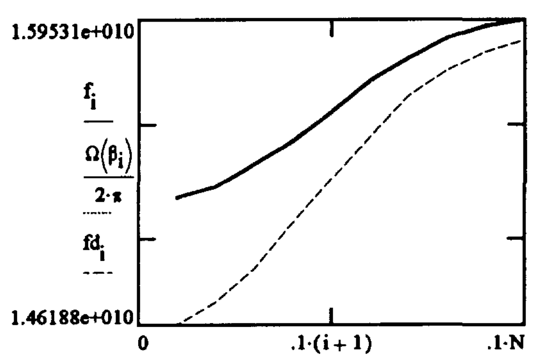
Calculate the phase velocity, group velocity, and equivalent beam voltage

$vp(i) := \frac{\Omega(\beta_i)}{\beta_i} \qquad vg(i) := \Omega'(\beta_i) \qquad V(v) := \left[\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^{-1} - 1\right] \cdot m \cdot \frac{c^2}{q}$

Calculate the interaction impedance and gap factor

$Z(i) := \frac{1}{(\beta_i)^2 \cdot vg(i)} \cdot \frac{\frac{\delta f(i)}{f_i}}{(\varepsilon r - 1) \cdot \varepsilon \cdot \pi \cdot r^2} \qquad M(i) := \left[\frac{\sin\left(\beta_i \cdot \frac{s}{2}\right)}{\beta_i \cdot \frac{s}{2}}\right]^2$

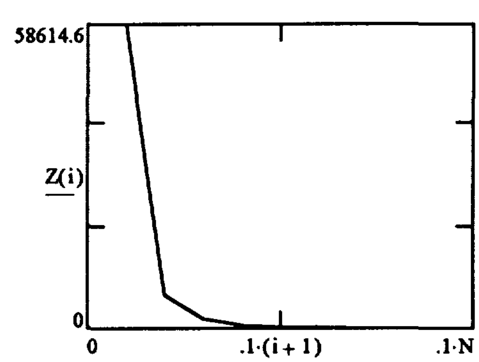
Plot of frequency, fit, and perturbed frequency



Values of frequency, fit, and interaction impedance

$i+1$	$f_i \cdot 10^{-9}$	$\Omega(\beta_i) \cdot \frac{10^{-9}}{2 \cdot \pi}$	$Z(i)$
1	15.178125	15.175524	58614.570394
2	15.228125	15.232496	6627.354587
3	15.325	15.321675	1942.569347
4	15.425	15.432365	726.471087
5	15.55625	15.553868	363.295224
6	15.684375	15.675488	216.833099
7	15.784375	15.786528	124.327911
8	15.875	15.876289	111.09255
9	15.928125	15.934076	139.059421
10	15.953125	15.949191	-344.364848

Plot of interaction impedance



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Thomas M. Wallett and A. Haq Qureshi

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13. ABSTRACT ***(Maximum 200 words)***
A disk-loaded circular wave guide structure and test fixture were fabricated. The dispersion characteristics were found by theoretical analysis, experimental testing, and computer simulation using the codes ARGUS and SOS. Interaction impedances were computed based on the corresponding dispersion characteristics. Finally, an equivalent circuit model for one period of the structure was chosen using equivalent circuit models for cylindrical wave guides of different radii. Optimum values for the discrete capacitors and inductors describing discontinuities between cylindrical wave guides were found using the computer code TOUCHSTONE.

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